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SHORTER COMMUNICATION

ON THE EXISTENCE OF THERMOCONVECTIVE ROLLS, TRANSVERSE TO A SUPERIMPOSED MEAN POISEUILLE FLOW

J.-M. LUIJKX and J. K. PLATTEN
University of Mons, Faculty of Sciences, Department of Thermodynamics,
7000 Mons, Belgium

and

J. CL. LEGROS
University of Brussels, Faculty of Applied Sciences,
Department of Physical Chemistry, E.P., 1050 Brussels, Belgium

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NOMENCLATURE

d,	depth of the fluid layer;
i_c^2,	$= -1$;
k_x,	$(\equiv \alpha)$ wave-number in the x-direction;
k_y,	wave-number in the y-direction;
l,	width of the fluid layer;
L,	length of the fluid layer;
Pr,	Prandtl number;
Q,	flow rate of the fluid $= \langle u \rangle ld$;
Ra,	Rayleigh number;
Ra'', $(Ra^\perp)$,	critical Rayleigh number corresponding to the onset of convection under the form of longitudinal (transverse) rolls;
Re,	Reynolds number using $\langle u \rangle$ as reference velocity;
Re^*,	'critical' Reynolds number (for which $Ra'' = Ra^\perp$);
$\langle u \rangle$,	mean velocity of the fluid;
v_z,	velocity component in the z-direction of a perturbation;
$\hat{W}(z)$,	amplitude of v_z;
x, y, z,	cartesian coordinates (see Fig. 1);
α,	$(\equiv k_x)$ wave-number in the x-direction;
$\beta = l/d$,	aspect ratio;
ΔT,	temperature difference between top and bottom of the fluid layer);
κ,	thermal diffusivity;
$\lambda = \pi/\alpha$,	length of a Bénard cell;
ν,	kinematic viscosity;
σ,	$= \sigma_R + i_c\sigma_I$: complex growth rate of a perturbation.

1. INTRODUCTION

ALMOST ten years ago, one of us (J.K.P.) published 'a preliminary experimental investigation of the stability of flows with an imposed temperature gradient' [1]. In that work, using heat flow measurements, experimental evidence was given that the critical Rayleigh number (corresponding to the onset of free convection) is an increasing function of the Reynolds number, measuring the Poiseuille velocity flow. These results, as well as other experimental findings (Legros *et al.* [2, 3] and Klapitz and Weill [4]) on the stability of flows in multi-component systems heated from below, are not clearly understood yet. The present paper is a new contribution to the study of the stability of a Poiseuille flow heated from below, i.e. to the Rayleigh Bénard problem with a superimposed basic laminar flow.

In an infinite duct (Fig. 1) bounded by two horizontal rigid and conducting plates located at $z = 0$ and $z = d$, a liquid flows in the x-direction under the effect of a horizontal pressure gradient (Poiseuille flow). The system is heated from below and, in the absence of Poiseuille flow, free convection, with roll-pattern corresponding to a structure with two non-vanishing velocity components, is expected when the Rayleigh number exceeds its critical value 1708. For longitudinal rolls ($v_z = \hat{W}(z)\, e^{ik_y y}$), it is well known that the critical Rayleigh number is unaffected by the flow [5, 6], but for transverse rolls ($v_z = \hat{W}(z)\, e^{ik_x x}$) the critical Rayleigh number increases with the Reynolds number [see Fig. 2(a)].

Usually, it is concluded that, when free convection occurs, the rolls must be longitudinal and that the mean flow has no influence on the onset of the secondary flow. However, this is only true in an infinite layer, and thus, this theoretical result cannot be used to understand completely the experimental findings of [1–4]. In real containers (of finite size), it is also well known [7, 8], that, in the absence of Poiseuille flow and above the critical point, the rolls are aligned parallel to the shorter vertical sides of the box. In this paper, we will not give an important significancy to the difference between the Davis' [7] 'finite rolls' calculations and the more realistic three-dimensional calculations by Davies-Jones [8].

In an infinite channel of rectangular cross-section, (Fig. 1), following the calculations by Davis or Davies-Jones, the critical Rayleigh number corresponding to finite transverse rolls must be smaller than the one corresponding to infinite longitudinal rolls. [Fig. 2(b) $Ra^\perp < Ra''$]. If a Poiseuille flow is imposed in the x-direction, characterised by its Reynolds number Re, Ra'' is independent of Re (no effect of the Poiseuille flow on infinite longitudinal rolls); on the other side, $Ra^\perp$ increases when Re increases (a stabilizing effect of the Poiseuille flow is expected when the disturbances are in the form of transverse rolls). Therefore, when looking at the smallest critical Rayleigh number, we conclude that, if the Poiseuille velocity is small enough ($Re < Re^*$), rolls are aligned parallel to the 'shorter' sides, with their axis perpendicular to the flow direction (the effect of the side walls is dominant). When the shear exceeds some critical value ($Re > Re^*$) convection occurs under the form of longitudinal rolls: this time, the effect of the flow is dominant. Thus, Re^* is a particular Reynolds number at which transverse rolls become less stable than longitudinal rolls. This number Re^* is a function of the aspect ratio $\beta = l/d$ and of the Prandtl number $Pr = \nu/\kappa$. The aim of the hydrodynamic stability theory is to determine $Ra^\perp = Ra^\perp(Re, \beta, Pr)$ and $Ra'' = Ra''(\beta)$ and thus

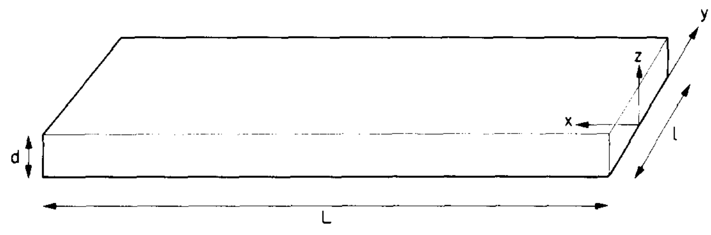


FIG. 1. System of coordinates. In the 'infinite' problem, $L \to \infty$, $1 \to \infty$.

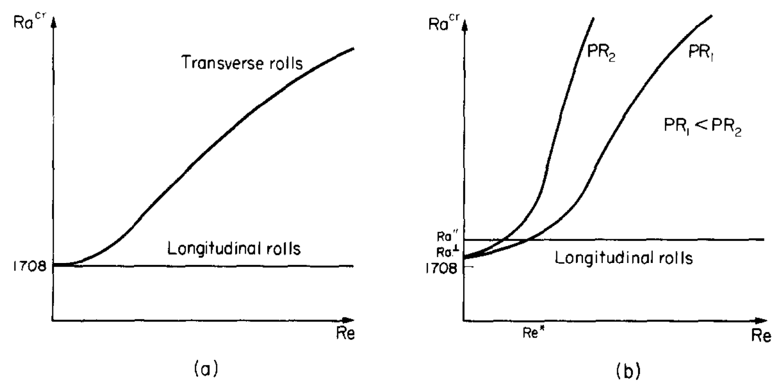


FIG. 2. Critical Rayleigh number vs Reynolds number. (a) Convection always occurs under the form of longitudinal rolls. (b) If $Re < Re^*$, convection occurs under the form of transverse rolls. Re^* decreases when Pr increases.

Re^* (β, Pr). This work is now in progress and is to be published shortly [9].

In the present work, we experimentally prove that transverse thermoconvective rolls exist when $Re < Re^*$. At higher values of the Reynolds number, the existence of longitudinal rolls seems obvious.

2. WORKING FLUID AND EXPERIMENTAL SET-UP

Silicone oil with kinematic viscosity of 0.5 $cm^2\ s^{-1}$ (Fluid 200 from Dow Corning) is used. Very regular patterns can easily be obtained with this component of high Prandtl number ($Pr \cong 450$) whose physical parameters do not vary rapidly with temperature. But, Re^* decreases when Pr increases [see Fig. 2(b)] and thus, with such a high Prandtl number fluid, very small shears have to be used in order to observe transverse rolls. From our numerical results [9], we found that with $Pr = 450$ and $\beta = 5$, $Re^* = O(10^{-2})$, i.e. a very small value indeed. However for $Pr = O(10)$ (e.g. water), $Re^* = O(1)$ and it is of some practical interest to accurately know the value of Re^* and the flow structure: indeed the thermal diffusion studies by the 'flow cell method' [2–4] are conducted in this range ($Re < 4$). Anyway, in this study, we shall restrict ourselves to silicone oil as working fluid in the range $Re \simeq 10^{-3}$–10^{-2}.

We built an observation cell which is shown as a schematic in Fig. 3. Classically, it is constituted by a rectangular frame in plexiglas, inserted between two thick (3 cm), polished copper plates. The temperatures of the plates are controlled by flow of thermoregulated water. The working volume is $1 \times 5.25 \times 93.5$ cm^3. In these conditions, $\beta = l/d = 5.25$ and this long cell looks like an infinite rectangular duct ($L/d \simeq 100$). Silicone oil is injected by gravity in this volume through two porous media parallel to the smaller sides. These porous media (from Porex, Glasrock, Fairburn) are 2 mm thick and high density polyethylene constituted, the pore sizes are 3.5×10^{-4} cm. The role of the first porous plate is to reduce the residual level of turbulence in the inlet part to a very low value, the second, located in the outlet part, is to conserve symmetry of the cell.

The convective structure which is eventually produced is observed by a shadowgraph method through the plexiglas

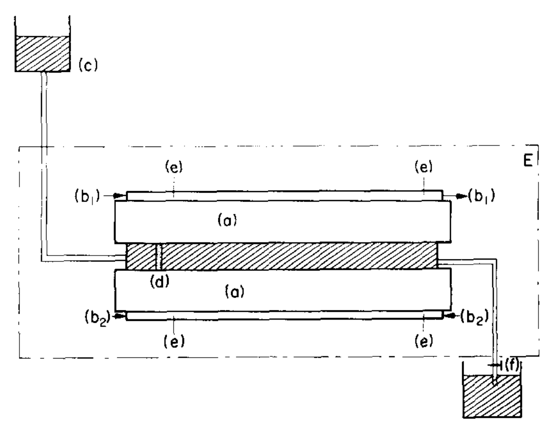


FIG. 3. Experimental set-up. (a) copper plates; (b1), (b2) flows of thermoregulated water; (c) reservoir; (d) porous medium; (e) temperature measurements; (f) valve; E: enclosure.

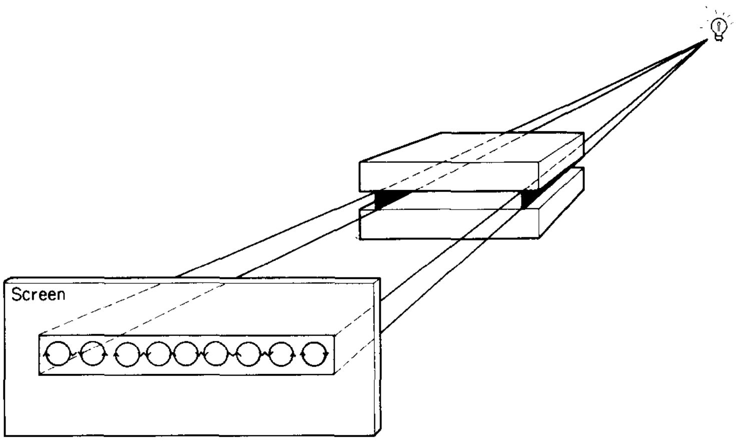
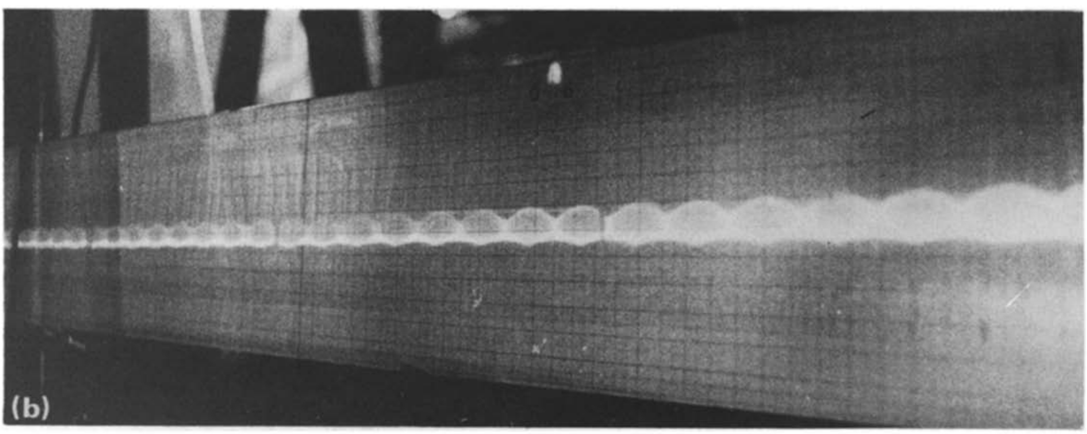


FIG. 4. Visualization of transverse convective rolls.

FIG. 4(a). Thermal lens effect: schema.

FIG. 4(b). Characteristic photograph.



walls by means of the thermal lens effect. A typical photograph of a part of a structure which can be observed near the critical point is shown in Fig. 4. It corresponds to very regular transverse rolls whose number is $O(10^2)$ ($k^{crit} \simeq 3.117$; $\lambda^{crit} \simeq 2\pi/3.117 \simeq 2$); thus one roll has a width $\lambda^{crit}/2$ equal to its height. In absence of free convection, or when the rolls are longitudinal, a view such as that given in Fig. 4 cannot be observed.

3. EXPERIMENTAL RESULTS

In all the experiments reported below (Fig. 5) (except in runs 0 and 7) we first impose a Poiseuille flow and after this we heat from below. Indeed if we first heat the system from below at $Re = 0$, inducing rolls parallel to the shorter sides, i.e. transverse rolls, and afterwards if we impose a Poiseuille flow, the eventually resulting transverse rolls, could be a consequence of these initial conditions.

Run 0: We first determine the critical temperature difference at which free convection sets in and we find $\Delta T^{crit} = 0.95°C \pm 0.03$. This yields a critical Rayleigh number $Ra^{crit} = 1711 \pm 55$ which is in perfect agreement with the theoretical value $Ra^{crit} = 1712$ for $\beta = 5.25$ [10]. In the 93.5 cm between the two porous walls, we observe 92 rolls. This leads to a critical wavenumber $k^{crit} = 3.09$ and this is

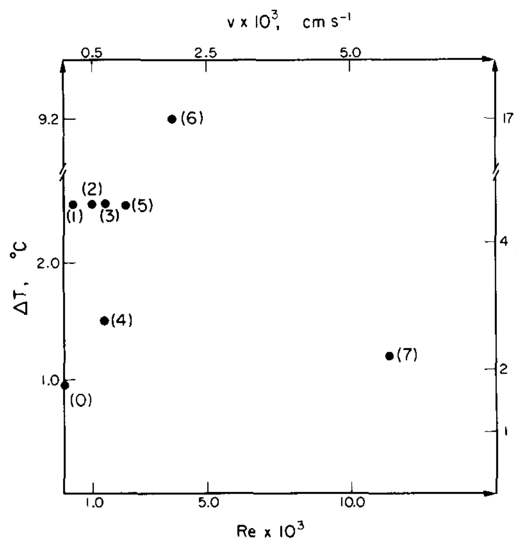


FIG. 5. Experiments. The runs' numbers are in parentheses.

also in agreement with the theory.

Of course, very near the critical point, the time needed to produce a regular roll pattern is of the order of two days. This is not convenient at all: in the presence of Poiseuille flow, the time needed to obtain a steady state must be much smaller than the residence time of the fluid in the duct. Therefore, we increase the temperature difference up to 2.5°C and we observe that 85 transverse rolls appeared. This corresponds to an increase of the wavelength which is also in agreement with other works (see for instance [11]).

Run 1: Total flow rate $Q = 210\,\mathrm{cm}^3$ in 67 h 59 m $= 8.6 \times 10^{-4}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ (estimated error 1%);

$$\text{mean velocity } \langle u \rangle = \frac{Q}{S} = \frac{8.6 \times 10^{-4}}{5.25} = 1.64 \times 10^{-4}\,\mathrm{cm\,s^{-1}};$$

$$\text{Reynolds number } Re = \frac{\langle u \rangle d}{\nu} = 0.328 \times 10^{-3};$$

temperature difference $\Delta T = 2.5°\mathrm{C}$.

After a few hours, we observe 85 transverse rolls which are travelling with a velocity greater than $\langle U \rangle$. They are stable: after three days the whole duct is still filled with the same number of rolls.

Run 2: $Q = 2.6 \times 10^{-3}\,\mathrm{cm^3\,s^{-1}}$;
$\langle u \rangle = 4.95 \times 10^{-4}\,\mathrm{cm\,s^{-1}}$;
$Re = 0.95 \times 10^{-3}$;
$\Delta T = 2.5°\mathrm{C}$.

We observed the formation of 87 *stable* transverse rolls.

These two runs clearly show that transverse rolls exist and are stable at small Reynolds numbers.

Run 3: $Q = 3.7 \times 10^{-3}\,\mathrm{cm\,s^{-1}}$;
$\langle u \rangle = 7.1 \times 10^{-4}\,\mathrm{cm\,s^{-1}}$;
$Re = 1.4 \times 10^{-3}$;
$\Delta T = 2.5°\mathrm{C}$.

Transverse rolls are formed rapidly near the porous walls as soon as the destabilizing temperature gradient is imposed, but these rolls look unstable; after one day there remains only 29 transverse rolls in the last 40 cm of the duct (the outlet part) and no structure was visible in the first 60 cm of the duct (the inlet part).

Run 4: Same as run 3 but $\Delta T = 1.5°\mathrm{C}$. After one day, there are transverse rolls everywhere, except in the first 12 cm of the duct; but after three days the region without visible structure has grown till to 34 cm.

Therefore, the rolls are probably less unstable than in run 3.

Run 5: $Q = 5.5 \times 10^{-3}\,\mathrm{cm\,s^{-1}}$;
$\langle u \rangle = 1.05 \times 10^{-3}\,\mathrm{cm\,s^{-1}}$;
$Re = 2.1 \times 10^{-3}$;
$\Delta T = 2.5°\mathrm{C}$.

Once more, rolls are formed rapidly near the porous boundaries but no more structure was visible in the duct the day after. We *suppose* the presence of longitudinal rolls, but this is still to be demonstrated.

These latest runs show that if the shear is increased, transverse rolls are more and more difficult to be formed, but this situation is not completely understood yet. Indeed the pattern depends not only on the Reynolds number, but also on the distance from the critical point ($Ra - Ra^{\mathrm{crit}}$) and the available theory is only valid for $Ra = Ra^{\mathrm{crit}}$. This is illustrated by the following runs.

Run 6: $Re = 3.7 \times 10^{-3}$; $T = 9.2°\mathrm{C}$. The distance from the critical point has been increased up to $Ra = 10Ra^{\mathrm{crit}}$ and the shear is also increased. We still observed transverse rolls but with a completely different size: $\lambda = 0.8\lambda^{\mathrm{crit}}$ (λ^{crit} is the critical wavelength corresponding to $Ra = Ra^{\mathrm{crit}}$ and $Re = 0$). A linear hydrodynamic stability theory cannot account for such a large effect.

Let us remark that all the preceding runs were performed with $Re < Re^*$ ($Re^* = 6.4 \times 10^{-3}$ for $Pr = 450$ and $\beta = 5.25$).

Run 7: $Re = 11.2 \times 10^{-3} > Re^*$; $\Delta T = 1.2°\mathrm{C}$. In this last run, we first impose $\Delta T = 1.2°\mathrm{C}$, inducing steady transverse rolls, which were rapidly pushed out the cell when the Poiseuille flow was imposed. Because the porous walls it is not possible to observe longitudinal structures in the x-direction. However, in run 7 we have an indirect proof of the existence of such longitudinal rolls. Indeed, we first stop the Poiseuille flow; no transverse structure appeared again, even after 24 h. After stopping the thermal gradient during 4 h and imposing it back at its initial value, transverse rolls appeared again in the whole apparatus with $\lambda = \lambda^{\mathrm{crit}}$. Indirectly this proves the existence of longitudinal rolls at $Re = 11.2 \times 10^{-3}$ and $\Delta T = 1.2°\mathrm{C}$: when the flow is stopped, these longitudinal rolls play the role of an initial condition of finite amplitude, in an experiment at $Re = 0$ and $\Delta T = 1.2°\mathrm{C}$. This initial condition prevails over the boundary conditions and inhibits the formation of transverse rolls. If we stop heating during 4 h, the presumed longitudinal rolls are destroyed and therefore, after the reset of the thermal gradient, transverse rolls have to be obtained back.

The last question to be discussed is the velocity at which the transverse rolls are convected by the Poiseuille flow. The mean velocity is given by $\langle u \rangle = Q/S$. Without changing the flow rate Q, once the rolls are formed, it is easy to follow the image of a particular roll on a screen and to determine the velocity U_{roll} at which this roll is convected. We found that this velocity U_{roll} is greater than $\langle u \rangle$, namely

$$U_{\mathrm{roll}} = 1.38 \times \langle u \rangle.$$

A possible explanation is the following: in the framework of a linear stability analysis, one should write for the z-component of the velocity perturbation of transverse rolls

$$v_z(x, y, z, t) = \hat{W}(y, z)\, e^{(i_c \alpha x - \sigma t)}. \tag{1}$$

At the critical point, the real part of σ is vanishing and therefore

$$v_z(x, y, z, t) = W(y, z)\, e^{i_c(\alpha x - \sigma_I t)}. \tag{2}$$

In our numerical study [9] we fitted the following empirical law

$$\sigma_I \simeq 1.5\alpha Re Pr. \tag{3}$$

Remembering that in equation (2) all the variables are dimensionless and that the scaling factor for the time is d^2/κ, this yields

$$v_z(x, y, z, t) = \hat{W}(y, z)\, e^{i_c \alpha (x - 1.5\langle u \rangle t)} \tag{4}$$

in which all the variables are dimensional. Equation (4) is the equation of a travelling wave whose velocity (which should be the velocity of the rolls system) is $1.5\langle u \rangle$. This result should be compared with the experimental value of $U_{\mathrm{roll}} = 1.38\langle u \rangle$ obtained at $Ra \simeq 2.5\, Ra^{\mathrm{crit}}$.

4. CONCLUSION

We proved the existence of stable transverse rolls at small Reynolds numbers and for higher shear values we have an indirect proof that the rolls are parallel to the flow direction. An accurate experimental determination of Re^* is not very easy and is now undertaken for different height to width ratios and for different fluids in order to study the influence of both Prandtl number and aspect ratio.

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REFERENCES

1. J. K. Platten and M. Lefebvre, A preliminary experimental investigation of the stability of flows with an imposed temperature gradient, *Physica* **51**, 330–332 (1971).
2. J. C. Legros, D. Rasse and G. Thomaes, Thermal Diffusion in a flowing layer with normal and adverse temperature gradients in the CCl_4–C_6H_6 system, *Physica* **57**, 585–593 (1972).
3. P. Poty, J. C. Legros and G. Thomaes, Thermal diffusion in some binary liquid mixtures by the flowing cell method, *Z. Naturforsch.* **29**, 1915–1916 (1974).
4. C. Klapitz and M. E. Weill, Thermodiffusion et stabilité en écoulement laminaire *J. Physique* **C1**, 145–149 (1976).
5. W. Velte, Zür Stabilität der Strömung in einem horizontalen Rohr bei ungleichmässig erwärter Wand, *Z. Angew. Math. Phys* **13**, 591–600 (1962).
6. R. Sani, Note on flow instability in heating ducts, *Z. Angew. Math. Phys* **15**, 381–387 (1964).
7. S. H. Davis, Convection in a box: linear theory, *J. Fluid Mech.* **30**, 465–478 (1967).
8. R. P. Davies-Jones, Thermal convection in an infinite channel with no-slip sidewalls, *J. Fluid Mech.* **44**, 695–704 (1970).
9. J.-M. Luijkx and J. K. Platten, Onset of free convection in a rectangular channel submitted to a laminar flow: a new critical Reynolds number (to be published).
10. J.-M. Luijkx and J. K. Platten, On the onset of free convection in a rectangualr channel, *J. Non-Equilib. Thermodyn* **6**(3), (1981).
11. E. L. Koschmieder, *Advances in Chemical Physics*, Vol. 26 pp. 194–200. John Wiley, New York (1974).